Seminar on the Social Discount Rate
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The problem of the discount rate and sustainable development

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Preliminary comments

- Uncertainties about our collective destiny.
- It's our duty to value these risks to advice our people about how much effort should be undertaken to improve the welfare of future generations.
- Let's do that far from the turmoil of politics, industrial lobbies, and green activists (normative approach).
- Two crucial problems to tackle:
  - Risk and uncertainty about the consequences of emitting CO_2;
  - Discounting.

Copenhagen Consensus

Project rating		Challenge	Opportunity
Very Good	1	Diseases	Control of HIV/AIDS
	2	Malnutrition	Providing micro nutrients
	3	Subsidies and Trade	Trade liberalisation
	4	Diseases	Control of malaria
Good	5	Malnutrition	Development of new agricultural technologies
	6	Sanitation & Water	Small-scale water technology for livelihoods
	7	Sanitation & Water	Community-managed water supply and sanitation
	8	Sanitation & Water	Research on water productivity in food production
	9	Government	Lowering the cost of starting a new business
Fair	10	Migration	Lowering barriers to migration for skilled workers
	11	Malnutrition	Improving infant and child nutrition
	12	Malnutrition	Reducing the prevalence of low birth weight
	13	Diseases	Scaled-up basic health services
Bad	14	Migration	Guest worker programmes for the unskilled
	15	Climate	Optimal carbon tax
	16	Climate	The Kyoto Protocol
	17	Climate	Value-at-risk carbon tax

Note to table: Some of the proposals were not ranked (see text below)

Discounted marginal damage of one tCO2

	Discount rate	Social value of CO2
Nordhaus	5%	8 $/tC02
Stern/Hope	1.4%	85 $/tC02

- Price of permits on the ETS market in 2009: 14€/tCO2.
- Carbon tax in France: 17€/tCO2.
- Cost of abatement:
  - Solar : 600-1200 €/tCO2
  - Wind: 25-200 €/tCO2

At which rate should we discount distant benefits?

- Consider time horizons exceeding those that are standard on financial markets: …. > 30 years
  1. The return of productive capital is uncertain.
  2. There is no observable risk free rate.
  3. The intertemporal MRS is difficult to measure, because c_t is uncertain.
- We need a model!
  - Ramsey (1928), CCAPM: Exogenous process for 3.
  - Weitzman (1998,…), Gollier & Weitzman (2010): Exogenous process for 1.

The 3 determinants of the discount rate

- Pure preference for the present/ethical attitude towards future generations (+)

- Preference for consumption smoothing over time + positive growth of GDP per capita (+): the marginal utility of 1 unit of consumption next period is smaller than the marginal utility of 1 unit of consumption now.

- Prudence (Kimball (1990)) + uncertain growth (-)

The term structure of the discount rate

- Is it socially efficient to reduce the discount rate for longer time horizons?

- This would favor the distant future relative to the short run.

- A potential argument:
  - more distant futures are more uncertain.
  - Under prudence, it has a negative effect on the discount rate.
  - But this is potentially counterbalanced by the fact that more distant generations are also wealthier on average.

- Serial correlations in growth rates are important.

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The extended Ramsey model

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The model

- Assume that LT insurance markets are efficient.
- => There is a representative agent in the economy.
- Standard Discounted Expected Utility SWF:

$$SWF = E\left[\int e^{-\delta t} u(c(t))dt\right]$$

- Consider a simple risk-free investment project with only one benefit occuring at date t. It equals $exp(rt)$ per dollar invested today.
- The discount rate associated to maturity t is the critical r such that a marginal investment in the project has no effect on the SWF

The pricing formula

- The discount rate is the IRR r_t such that the investment does not affect social welfare at the margin:

$$SWF = E\left[\int e^{-\delta t} u(c(t))dt\right]$$

$$e^{r_t t} e^{-\delta t} Eu'(c_t) = u'(c_0)$$

$$\boxed{r_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_t)}{u'(c_0)}}$$

$$u'(c) = c^{-\gamma}$$

$$c_t = c_0 e^{gt}$$

$$\boxed{r_t = \delta - \frac{1}{t}\ln e^{-\gamma g t} = \delta + \gamma g}$$

The role of the convexity of *u'*

The extended Ramsey rule in the iid lognormal case

$$c_{t+1} = c_t e^{x_t}, \text{ with } x_t \text{ i.i.d.} \sim N(\mu, \sigma)$$

$$g = \ln(Ec_1 / c_0) = Ee^x = \mu + 0.5\sigma^2$$

$$\frac{Eu'(c_t)}{u'(c_0)} = \frac{E\left[c_0^{-\gamma} \prod_\tau e^{-\gamma x_\tau}\right]}{c_0^{-\gamma}} = \left[Ee^{-\gamma x}\right]^t = \left[e^{-\gamma(\mu - 0.5\gamma\sigma^2)}\right]^t.$$

$$r_t = \delta - \frac{1}{t}\ln\frac{Eu'(c_t)}{u'(c_0)} = \delta + \gamma\mu - 0.5\gamma^2\sigma^2.$$

$$r = \delta + \gamma g - 0.5\gamma(\gamma + 1)\sigma^2.$$

Impatience — wealth effect — precautionary effect

Estimation of inequality aversion

- Consider an economy with 2 social groups of equal size, A and B. Each agent in group A is 2 times wealthier than in group B.
- We can transfer wealth from A to B. What is the maximum sacrifice of A that Society should accept for B to get one more dollar ?

γ	MRS 2 wA = 2*wB	MRS 10 wA = 10*wB
0	1,00	1,00
0,5	1,41	3,16
1	2,00	10,00
1,5	2,83	31,62
2	4,00	100,00
4	16,00	10000,00

Standard time-series calibration of the extended Ramsey rule

- Kocherlakota (1996), using United States annual data over the period 1889-1978, estimated the standard deviation of the growth of consumption per capita to 3.6% per year.

$$\sigma^2 = (0.036)^2 \text{ and } \gamma = 2 \text{ implies } 0.5\gamma(\gamma+1)\sigma^2 = 0.4\%.$$

- Benchmark calibration

g	σ	δ	γ
2%	*3.6%*	*0%*	*2*

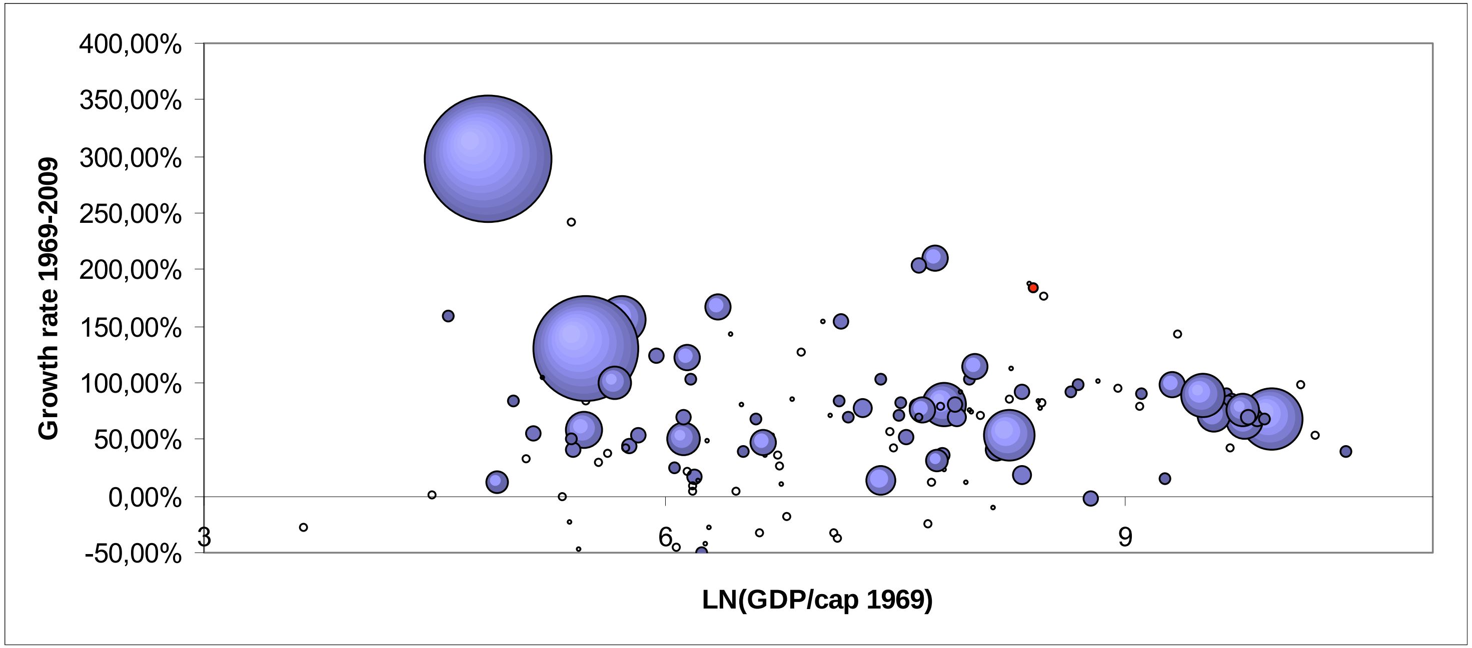
$$r = \delta + \gamma g - 0.5\gamma(\gamma+1)\sigma^2 = 3.6\%$$

Calibration of a random walk for the growth rate, using country-specific data 1970-2010

Country	discount rate
United States	3,37 %
Sweden	3,49 %
Germany	3,45 %
United Kingdom	3,60 %
Japan	4,51 %
China	15,36 %
South Korea	10,68 %
Taiwan	10,10 %
India	6,71 %
Former Soviet Union	1,70 %
Gabon	-0,23 %
Liberia	-15,94 %
Zaire (RDC)	-6,23 %
Zambia	-1,84 %
Zimbabwe	-1,81 %

Alternative cross-sectional calibration of the extended Ramsey rule

- 190 countries over the period 1969-2009:



μ	σ	δ	γ
1.5%	*11%*	*0%*	*2*

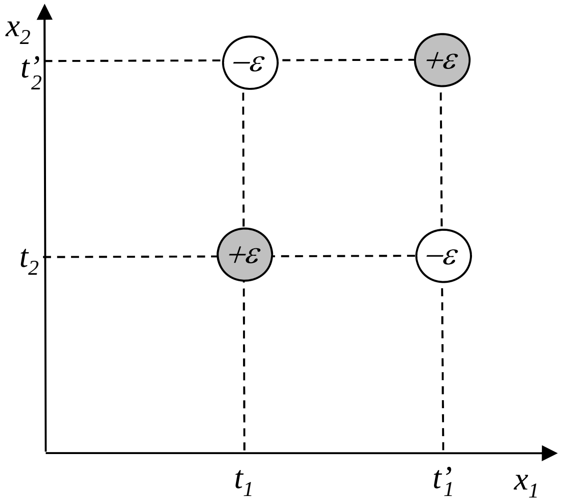
$$r = \delta + \gamma g - 0.5\gamma(\gamma+1)\sigma^2$$
$$= 0\% + 3\% - 2.42\%$$
$$= 0.58\%$$

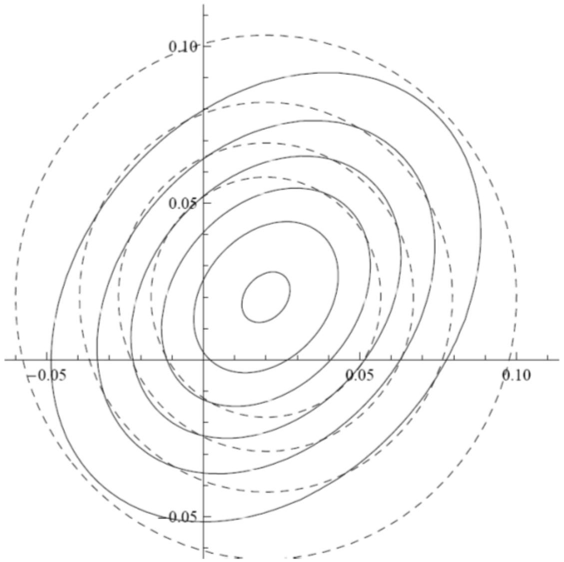
The main dish: Decreasing term structure

- When the growth rate of c is a random walk, the annualized wealth and precautionary effects are constant.
- We may question the absence of serial correlation.
- Suppose alternatively that there is some *positive concordance* (Tchen (1980), Epstein and Tanny (1980)) is the distribution of (x_1, x_2).
- This magnifies the LT risk. If $u'(\exp(x_1 + x_2))$ is SPM, this yields a decreasing term structure.
- In the remainder of this presentation, I will illustrate this with a few calibrated stochastic models:
  - Mean-reverting AR(1)
  - Markov switches
  - Parameter uncertainty.

$$r_t = \delta - \frac{1}{t} \ln \frac{Eu'(c_t)}{u'(c_0)}$$

Positive concordance





$$x_t = \phi x_{t-1} + (1-\phi)\mu + \varepsilon_t$$

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AR(1)

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A simple extension: Mean-reverting growth process

$$\begin{cases} c_{t+1} = c_t e^{x_t} \\ x_t = \phi x_{t-1} + (1-\phi)\mu + \varepsilon_t \\ \varepsilon_1, \varepsilon_2, \ldots i.i.d. \sim N(0,\sigma) \end{cases}$$

- Persistence. Suppose $x_{-1}=\mu$, and $\varepsilon_0>0$. It implies that

$$Ex_t = \mu + \phi^t \varepsilon_0$$

- We have that

$$\ln c_t - \ln c_0 = \mu t + (x_{-1} - \mu)\frac{\phi(1-\phi^t)}{1-\phi} + \sum_{\tau=1}^{t} \frac{1-\phi^\tau}{1-\phi}\varepsilon_{t-\tau} \sim Normal$$

$$r_t = \delta - \frac{1}{t}\ln E\left[e^{-\gamma(\ln c_t - \ln c_0)}\right].$$

$$r_t = \delta + \gamma t^{-1} E\left[\ln c_t - \ln c_0\right] - 0.5\gamma^2 t^{-1} Var(\ln c_t).$$

$$r_t = \delta + \gamma\mu - 0.5\frac{\gamma^2\sigma^2}{(1-\phi)^2} + \gamma\frac{\phi(1-\phi^t)}{t(1-\phi)}\left[x_{-1} - \mu + 0.5\gamma\sigma^2 \frac{2 - \frac{\phi(1+\phi^t)}{1+\phi}}{(1-\phi)^2}\right].$$

Term structure and the evolution of the short-term rate

$$r_1 = \delta + \gamma\mu - 0.5\gamma^2\sigma^2, \quad (x_{-1} = \mu) \quad \Rightarrow \quad r_\infty = \delta + \gamma\mu - 0.5\frac{\gamma^2\sigma^2}{(1-\phi)^2}$$

- Mean-reversion reduces the rate at which very distant cash-flows must be discounted.
- Positive correlation in the *x* magnifies long term risk!
- Let $r_1(t)$ denote the rate that should be used at date t to discount cash flows occurring at date $t+1$. It follows the following AR(1):

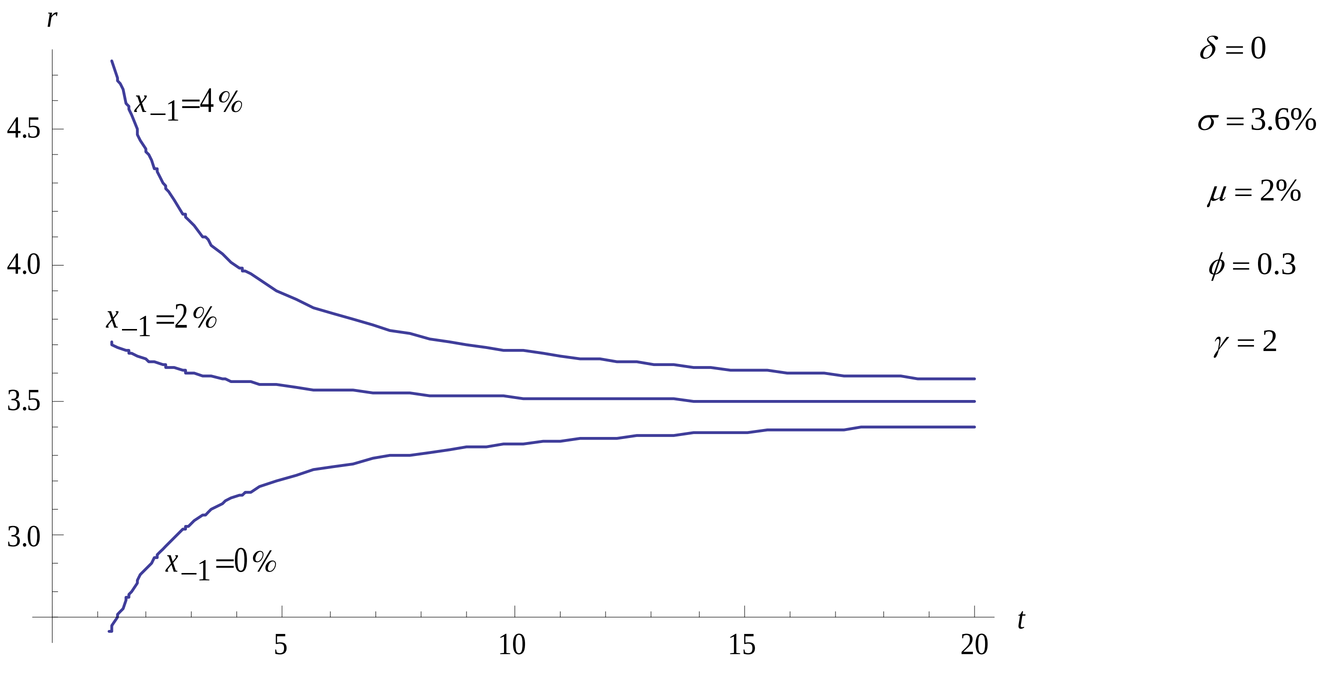
$$\begin{cases} r_1(t) = \delta - 0.5\gamma^2\sigma^2 + \gamma y_{t-1} \\ y_t = \phi y_{t-1} + (1-\phi)\mu + \gamma\varepsilon_t, \end{cases}$$

$$y_{t-1} = \phi x_{t-1} + (1-\phi)\mu$$

(Vasicek)

Calibration

- Huge literature on the term structure. Its basic model assumes an AR(1) for $r_1(t)$.
- Backus, Foresi and Telmer (1998) consider $\phi=0.024$ month^{-1}, which corresponds to $\phi=0.3$ year^{-1}: Half-life time of 2.3 years.



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Markov switches

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Two-regime Markov process

$$\begin{cases} c_{t+1} = c_t e^{x_t} \\ x_t = \mu^{s_t} + \varepsilon_t \\ P\left[s_{t+1} = b \middle| s_t = g\right] = \pi^g; \quad P\left[s_{t+1} = g \middle| s_t = b\right] = \pi^b \end{cases}$$

$$\frac{E\left[u'(c_1)\middle|s\right]}{u'(c_0)} = (1-\pi^s)Ee^{-\gamma(\mu^s+\varepsilon_0)} + \pi^s Ee^{-\gamma(\mu^{-s}+\varepsilon_0)} = e^{0.5\gamma^2\sigma^2}\left[(1-\pi^s)e^{-\gamma\mu^s} + \pi^s e^{-\gamma\mu^{-s}}\right].$$

$$r_1^s = \delta + \gamma m_1^s - 0.5\gamma^2\sigma^2,$$

$$e^{-\gamma m_1^s} = (1-\pi^s)e^{-\gamma\mu^s} + \pi^s e^{-\gamma\mu^{-s}}.$$

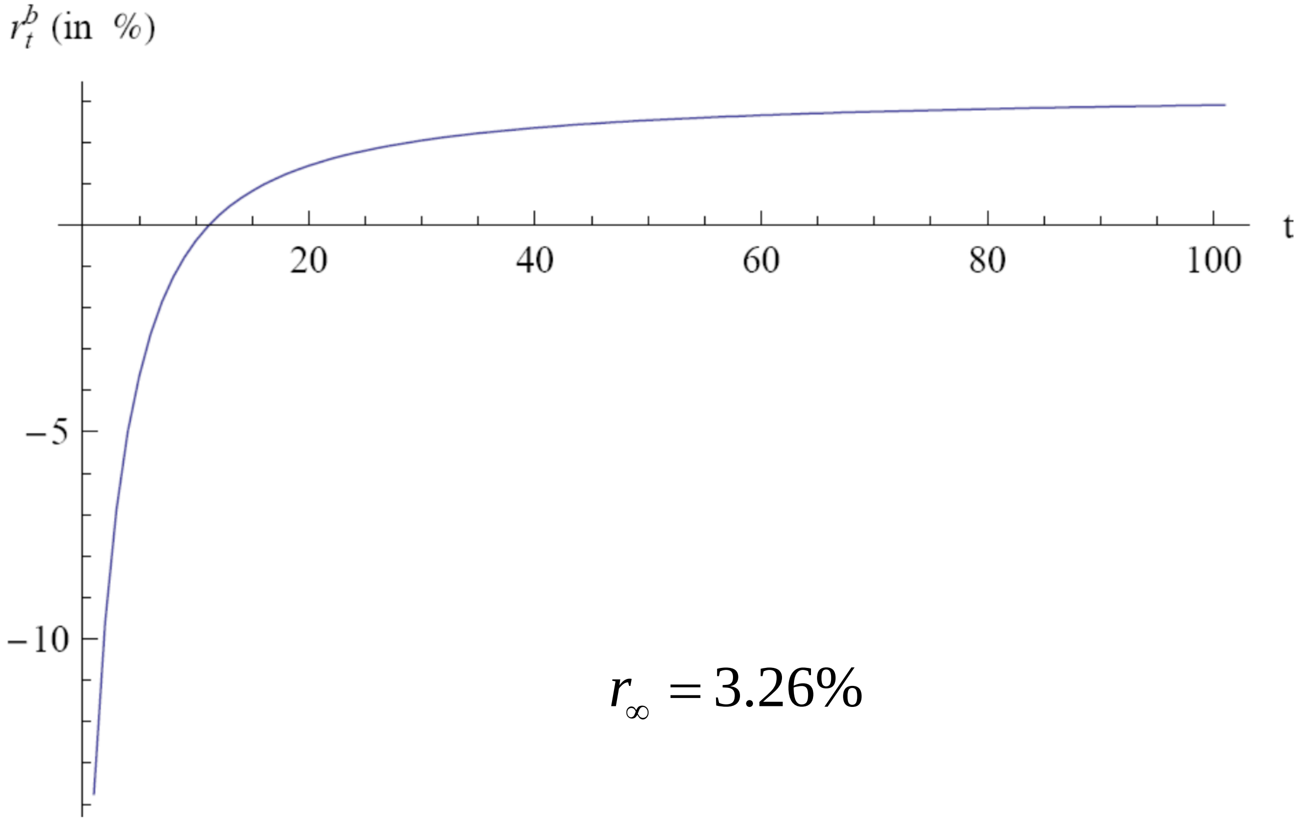
Precautionary equivalent growth rate

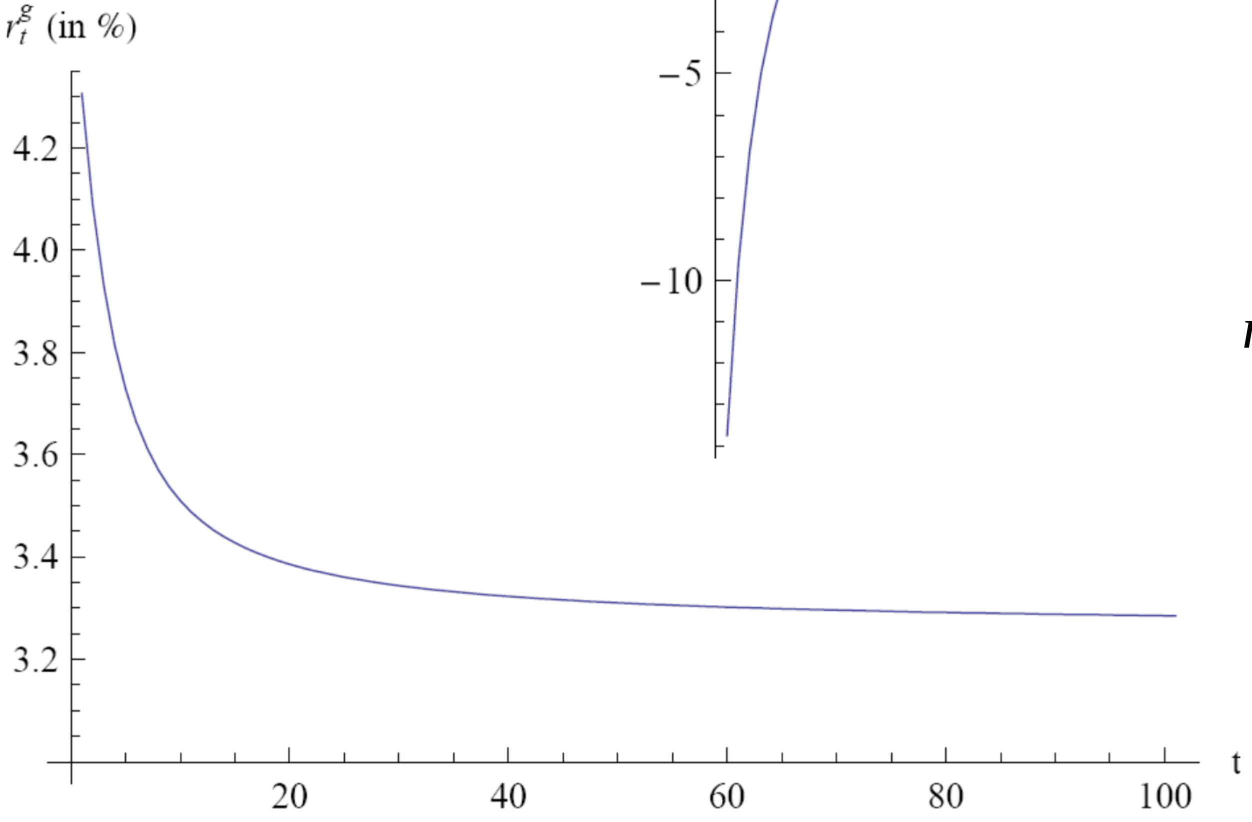
$$m_1^g \geq m_1^b$$

Numerical sim I

- Link with the literature on extreme events (Rietz (1988), Aase (1993), Barro (2006)).
- Cecchetti, Lam and Mark (2000) estimated a two-state regime-switching process for the US economy using the annual per capita consumption data covering the period 1890-1994.
- The unconditional expected growth rate is *1.89%*.

μ^g	μ^b	π^g	π^b	σ
2.25%	*−6.78%*	*2.2%*	*48.4%*	*3.13%*





$$r_\infty = 3.26\%$$

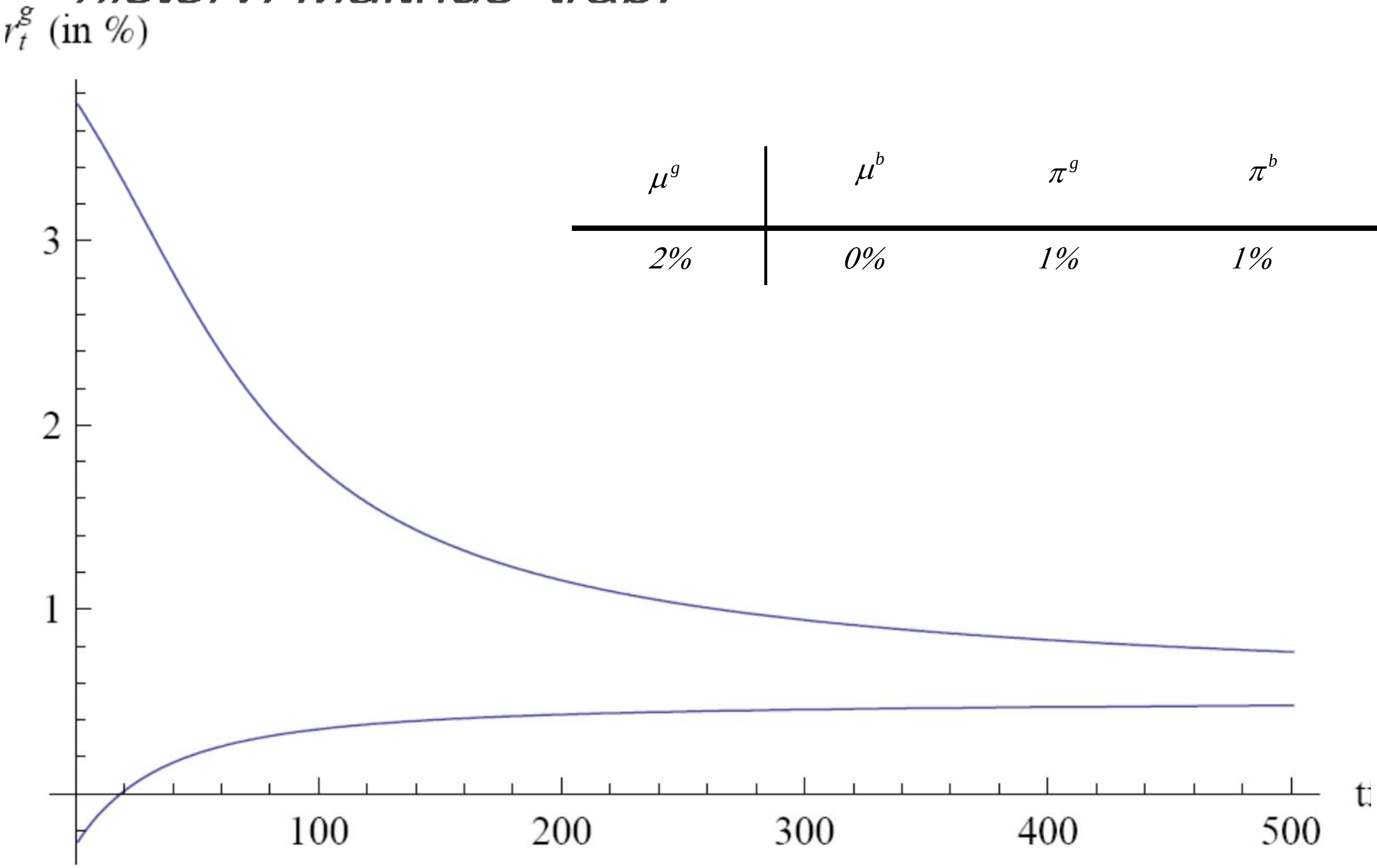
Persistent shocks on the growth rate

- Daily wage (in pounds of wheat):
  - In Babylon (1880-1600 B.C.): around 15;
  - In the golden age of Pericles in Athens: around 26;
  - In England around 1780: 13.
- Malthus Law? Stable 0% growth of GDP/cap.
- Switch to a trend of 2% around 1800-1850.

Numerical sim II

- The calibration based on data covering the period 1890-1994 fails to recognize a crucial aspect of economic history: Malthus’ trap.

μ^g	μ^b	π^g	π^b	σ
2%	0%	1%	1%	3.6%



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Parametric uncertainty and fat tails

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Uncertain growth

- Dynamic process on c_t parametrized by θ.
- $\theta=1,\ldots,n$ with probabilities $q_1,q_2,\ldots,q_n$.
- By the law of iterated expectations, we have that

$$Eu'(c_t) = \sum_{\theta=1}^{n} q_\theta E\left[u'(c_t)\middle|\theta\right].$$

$$r_t = \delta - \frac{1}{t}\ln\sum_{\theta=1}^{n} q_\theta \frac{E\left[u'(c_t)\middle|\theta\right]}{u'(c_0)} = -\frac{1}{t}\ln\sum_{\theta=1}^{n} q_\theta e^{-r_{t\theta}t}$$

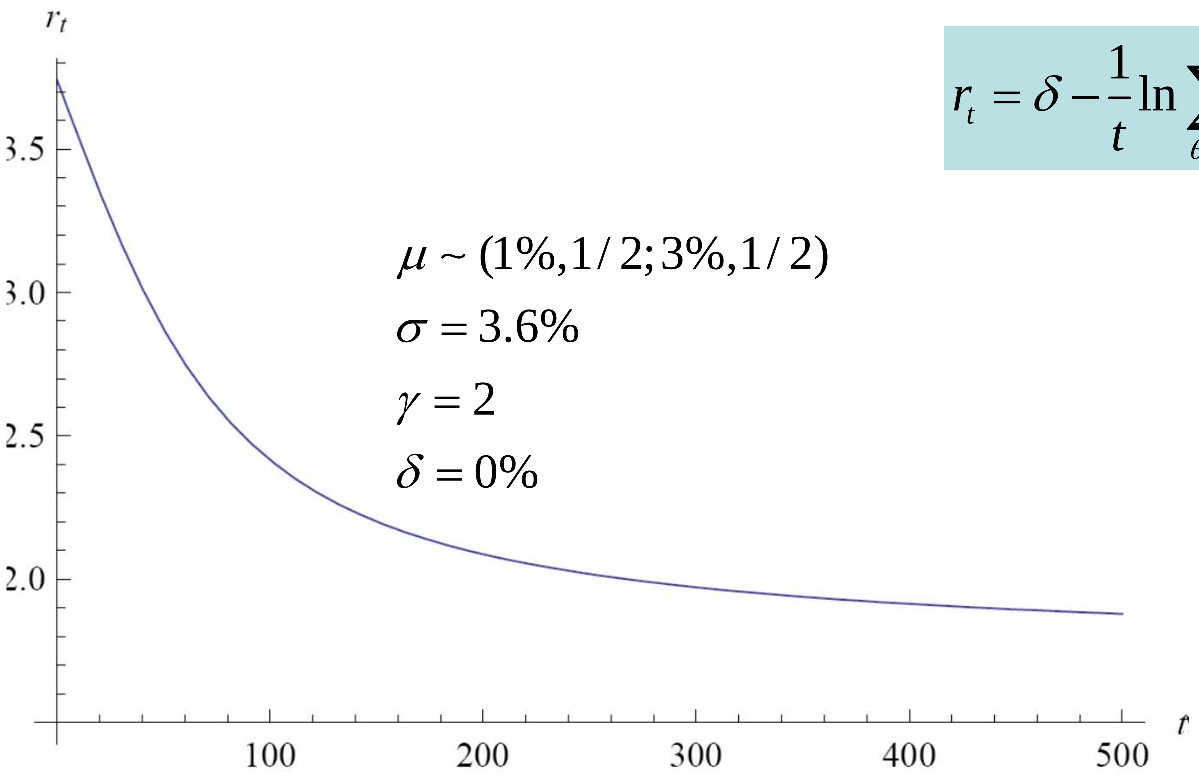
$$r_{t\theta} = \delta - \frac{1}{t}\ln\frac{E\left[u'(c_t)\middle|\theta\right]}{u'(c_0)}$$

Conditional to θ, the growth process is a random walk

$$\begin{cases} c_{t+1} = c_t e^{x_t} \\ x_0, x_1, \ldots \mid \theta \ i.i.d. \sim N(\mu_\theta, \sigma_\theta) \ \forall \theta \\ \theta \sim (1, q_1; \ldots; n, q_n) \end{cases}$$

$$r_{t\theta} = \delta + \gamma\mu_\theta - 0.5\gamma^2\sigma_\theta^2.$$

$$r_t = \delta - \frac{1}{t} \ln \sum_{\theta=1}^{n} q_\theta e^{(-\gamma\mu_\theta + 0.5\gamma^2\sigma_\theta^2)t}.$$



$\mu \sim (1\%, 1/2; 3\%, 1/2)$

$\sigma = 3.6\%$

$\gamma = 2$

$\delta = 0\%$

The case of an unknown trend of economic growth

- Suppose that σ is known, but μ is normally distributed with mean μ_0 and std deviation σ_0.

$$r_t = \delta - \frac{1}{t}\ln e^{(-\gamma\mu_0 + 0.5\gamma^2 t\sigma_0^2 + 0.5\gamma^2\sigma^2)t} = \delta + \gamma\mu_0 - 0.5\gamma^2(\sigma^2 + \sigma_0^2 t).$$

$$\left.\begin{array}{l} \ln \frac{c_t}{c_0}\Big|\mu,\sigma \sim N(\mu t, \sigma^2 t) \\ \mu t \sim N(\mu_0 t, \sigma_0^2 t^2) \end{array}\right\} \Rightarrow \ln\frac{c_t}{c_0} \sim N(\mu_0 t, \sigma^2 t + \sigma_0^2 t^2)$$

$$\min r_\theta = -\infty$$

The case of an unknown volatility of economic growth

- Weitzman (2007, 2009) : Suppose alternatively that μ is known, but σ is not.
- We work with the precision $p_\theta = \sigma_\theta^{-2} \sim \Gamma(a,b)$.
- Unconditional distribution of x_t:

$$\left.\begin{array}{c} x \,\middle|\, p \sim N(\mu, \sigma = 1/\sqrt{p}) \\ p \sim \Gamma(a,b) \end{array}\right\} \Rightarrow \frac{x-\mu}{1/\sqrt{ab}} \sim Student(2a)$$

- As is well-known also, this Student's t-distribution has fatter tails than the corresponding normal distribution with the same mean and variance.

$$r_t = \delta - \frac{1}{t} \ln \frac{Eu'(c_t)}{u'(c_0)} = -\infty$$

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Conclusion

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What we should do for the future

- Which future are we talking about?
- Wealth effect and precautionary effect.
- In the long run, the precautionary effect may dominate everything else.
- This is the case when shocks on the growth rate of the economy have some degree of persistency.
- Which discount rate? I used to advocate
  - 4% for the short term;
  - 2% for the distant future.
- This is a real rate, for a small risk free project…
- Recession in Europe: We should reduce the DR for short maturities.