

Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate*

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A critical feature of the distant future is currently unresolvable uncertainty about what will then be the appropriate rate of return on capital to use for discounting. This paper shows that there is a well-defined sense in which the “lowest possible” interest rate should be used for discounting the far-distant future part of any investment project. Some implications are discussed for evaluating long-term environmental projects or activities, like measures to mitigate the possible effects of global climate change. © 1998 Academic Press

1. INTRODUCTION

This paper concerns the proper social discounting of events that happen in the “distant future”—a term purposely left vague, but meaning, loosely, generations and even centuries from the present time. Increasingly today, we are being asked to analyze environmental projects or activities whose effects will be spread out over hundreds of years. Prominent examples include: global climate change, radioactive waste disposal, loss of biodiversity, thinning of stratospheric ozone, groundwater pollution, minerals depletion, and many others.

There is a “problem” with discounting the distant-future payoffs of projects or activities, which has been widely noted and commented upon. To think about the distant future in terms of standard discounting is to have an uneasy intuitive feeling that something is wrong, somewhere.

The logic of exponential discounting forces us to say that what we might otherwise conceptualize as monumental events “do not much matter” when they occur in future centuries or millennia. Perhaps even more disturbing than the *absolute* shrinkage of distant future events from exponential discounting is the *relative* shrinkage that occurs. By the logic of compound interest, the importance of a cataclysmic event happening *four* centuries from now should be *much less* significant for us today *relative to* the importance of a cataclysmic event occurring *three* centuries from now. Yet almost no one really feels this way about the distant future. Rather, we tend not to attribute much weight to whether an event occurs

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three or four centuries from now, a phenomenon sometimes called “empathetic distance.”¹

Few are the economists who have not sensed in their heart of hearts that something is amiss about treating a distant future event as just another term to be discounted away at the same constant exponential rate gotten from extrapolating past rates of return to capital. Indeed, there is experimental evidence that people generally discount the future at declining rates of interest.²

Responding to this kind of ambivalence have been various proposals to reconcile “normal” discount rates for the near future with “low” discount rates for the far future. The proposed mechanisms range from openly *ad-hoc* adjustments to formal axiomatic treatments.³

This paper is centered on what might be called a “basic reason” or a “generic argument” why events in the far-distant future should be discounted at the lowest possible rate. The basic logic is simple and yet, I believe, powerful and general. In a sense, the same relentless force of compound interest that gives rise to the paradox in the first place will act toward dissipating it when the very interest rate at which the discounting must occur is uncertain.

2. THE MODEL

This paper applies to a period called the “distant future,” whose features are hazy in our minds today. Yet, even so, we must now make investment decisions having consequences that will occur in this hazy future. How are these consequences to be weighed against each other?

While there is uncertainty about almost everything in the distant future, perhaps the most fundamental uncertainty of all concerns the discount rate itself. The question before us is how to discount this distant future in such a way as will induce us to make the best possible investment decisions now, in our present state of uncertainty about the relevant interest rate that will then apply.

We proceed as follows. Suppose there are n possible “scenarios” for how the future might unfold, indexed by $j = 1, 2, \dots, n$. In scenario j , the discount or interest rate at time t is

$$r_j(t). \tag{1}$$

As of now we do not know which scenario will occur, but we currently estimate that the probability of scenario j is

$$p_j, \tag{2}$$

where $p_j > 0$ and $\sum p_j = 1$.

¹ Schelling [13] contains a useful discussion of this issue.

² Ainslie [1]; Cropper, Aydede and Portney [6]. Although “hyperbolic discounting” typically refers to relatively short-run ingrained behavior, I believe that an evolutionary argument (it occurs in the real world because it had survival value and we witness only the survivors) could be given along the lines of the model of this paper.

³ Cline [5]; Azvar [3]; Henderson and Bateman [10]; Loewenstein and Prelec [11]; Chichilnisky [4]; Heal [9]. An interesting approach based on risk aversion about an uncertain growth rate is described in Arrow *et al* [2, p. 137] and modeled formally in a paper by Gollier [8].

The way we are conceptualizing the problem here is to have each $r_j(t)$ be close to today's interest or discount rate for small t , but approaching some other limit as time goes to infinity. Assuming the limit exists, let

$$r_j^* \equiv \lim_{t \rightarrow \infty} r_j(t). \quad (3)$$

Then we can say that the limiting interest rate will be r_j^* with probability p_j .

The distribution of long-run interest or discount rates is treated here as a given primitive. It is unnecessary to specify a full general-equilibrium model, since, for the purposes of this paper, what matters is the partial-equilibrium reduced-form limiting dispersion of discount rates, however it comes about.

There are many reasons why the far-distant-future interest rate might be considered to be a (nonmean-reverting) random variable from today's perspective. When I try to imagine how the future world might look a century from now, I start by trying to conceptualize how people a century ago might have attempted to envision our world today. We have available now some important technologies, like computers or airplanes, that were essentially unimaginable 100 years ago. Maybe a now unimaginable "photon-based technology" will replace today's electronic technology and deliver such prodigious rates of technological progress with a clean environment that historians then will look back on the previous 100 years and smile at the modest projections of even the growth optimists at the close of the twentieth century. Or, who knows, maybe a century from now people will feel crowded and polluted and very disappointed in a pace of technological change that failed to maintain the productivity growth of the "golden age" of the industrial revolution during the earlier two centuries from 1800 to 2000. It should be apparent that these two scenarios imply very different rates of return.

Behind the far-distant-future interest rate is the long run productivity of capital, which depends on a host of factors unknowable at the present time. There are fundamental uncertainties about the rate of economic growth, the amount of capital that will be accumulated, the degree of diminishing returns, the state of the environment, the state of international relations, the level and pace of technological progress, the rate of pure time preference, the degree of substitutability of accumulable for nonaccumulable factors, and all of the many other economic and noneconomic features that might be relevant to determining the distant-future own rate of return on consumption. Even if we thought of interest rates as being a mean-reverting random variable, our best statistical estimate of this mean is itself a random variable, which makes the reduced-form overall stochastic process be non-mean-reverting.⁴

The relevant "discount factor" for scenario j is

$$a_j(t) \equiv \exp\left(-\int_0^t r_j(\tau) d\tau\right) \quad (4)$$

meaning that an extra dollar at time t in scenario j is worth $a_j(t)$ dollars now.

What might be called the *certainty-equivalent discount factor* at time t is

$$A(t) \equiv \sum p_j a_j(t). \quad (5)$$

⁴ I owe this insight to Andrew Metrick.

The meaning of $A(t)$ can be explained as follows. Suppose that an investment choice with distant-future consequences must be made now, *before* we know what the relevant scenario will be. Suppose the investment is “small” relative to the overall size of the world economy and its uncertainty is uncorrelated with the state of the world.⁵ Then an extra dollar of the investment paid out or taken in at time t is worth today $A(t)$ expected present dollars.

By definition, the *certainty-equivalent instantaneous discount rate* corresponding to (5) is

$$R(t) \equiv -\frac{\dot{A}(t)}{A(t)}. \quad (6)$$

The *certainty-equivalent far-distant-future discount rate* is then defined to be

$$R^* \equiv \lim_{t \rightarrow \infty} R(t). \quad (7)$$

Note that it does not matter whether we think of R^* as an *average* or an *instantaneous* interest rate, since the two concepts coincide in the limit.

The main task of this paper is to determine the value of R^* and to understand what it depends upon.

3. THE BASIC RESULT

We are now ready to state the main proposition.

Define the *lowest-possible far-distant-future discount rate* to be

$$r_{\min}^* \equiv \text{minimum}_j \{r_j^*\}. \quad (8)$$

The only restriction being applied to the $\{r_j^*\}$ is that they each be nonnegative. Thus, it is permitted for r_{\min}^* to equal zero. The following proposition is the basic result of the paper.

PROPOSITION

$$R^* = r_{\min}^*. \quad (9)$$

Proof. Differentiating (5) with respect to time gives

$$\dot{A}(t) = \sum p_j \dot{a}_j(t), \quad (10)$$

which can be rewritten, using (4), as

$$\dot{A}(t) = -\sum p_j r_j(t) a_j(t). \quad (11)$$

⁵ The formulation can be modified if this assumption is not met, but the mathematics is more complicated while the basic point of the paper will remain.

Combining (11) with (5) and (6), we have the expression

$$R(t) = \frac{\sum p_j r_j(t) a_j(t)}{\sum p_j a_j(t)}. \quad (12)$$

Now rewrite (12) as the weighted-average formula

$$R(t) = \sum w_j(t) r_j(t), \quad (13)$$

where

$$w_j(t) \equiv \frac{p_j a_j(t)}{\sum p_j a_j(t)} > 0 \quad (14)$$

and

$$\sum w_j(t) = 1. \quad (15)$$

Without loss of generality suppose that scenario 1 has the unique lowest limiting interest rate, so that

$$r_1^* = \underset{j}{\text{minimum}} \{r_j^*\}. \quad (16)$$

Then, passing to the limit with (14), (4), (3) obtains, for $j \neq 1$, the result

$$\lim_{t \rightarrow \infty} \frac{w_j(t)}{w_1(t)} \left[= \lim_{t \rightarrow \infty} \frac{p_j \exp\left(-\int_0^t r_j(\tau) d\tau\right)}{p_1 \exp\left(-\int_0^t r_1(\tau) d\tau\right)} = \lim_{t \rightarrow \infty} \frac{p_j}{p_1} \exp\left(-(r_j^* - r_1^*)t\right) = 0. \right] \quad (17)$$

Applying (17) and (15) then yields the desired conclusion of (13) that

$$\lim_{t \rightarrow \infty} w_1(t) = 1, \quad (18)$$

while

$$\lim_{t \rightarrow \infty} w_j(t) = 0 \quad (19)$$

for $j \neq 1$. ■

4. DISCUSSION

The proposition tells us that the interest rate for discounting among events within the far distant future should be its lowest possible limiting value. From today's perspective, the only relevant limiting scenario is the one with the lowest interest rate—all of the other states at that far-distant time, by comparison, are relatively much less important now because their present value has been reduced by the power of compound discounting at a higher rate.

The key insight here is that what should be averaged over states of the world is not discount *rates* at various times, but discount *factors*. In the limit, the properly-averaged certainty-equivalent discount *factor* corresponds to the *minimum* discount *rate*.

To get a more intuitive feeling for what the main proposition signifies, suppose, as a kind of pure thought experiment, that we conceptualize the future as being divided sharply into two distinct regimes. Suppose the “near-future” regime from time zero (now) until time T is characterized by a deterministic interest rate whose known value is \bar{r} . The “distant-future” regime runs from time T until infinity and is characterized by currently uncertain interest rates that may assume the (constant) value r_j with probability p_j . Suppose that

$$\bar{r} = \sum p_j r_j. \quad (20)$$

Then the time profile of the certainty-equivalent instantaneous discount rate $R(t)$ —as viewed from the present time—will hold steady at level \bar{r} from now until time T , but thereafter will be declining continuously, down to the limiting value of $R^* = \min\{r_j\}$.

The monotone decline of $R(t)$ for $t > T$ is a straightforward consequence of its definition. Differentiating (13) with respect to time yields

$$\dot{R}(t) = \sum \dot{w}_j(t) r_j, \quad (21)$$

where, from (14),

$$\dot{w}_j(t) = w_j(t) \left(\sum_i w_i(t) r_i - r_j \right). \quad (22)$$

Combining (21) with (22) yields the basic equation

$$\dot{R}(t) = - \sum w_j(t) (r_j - R(t))^2, \quad (23)$$

which must be negative.⁶

In this context, therefore, the following two statements are not contradictory. First, it is accurate to say that the force of compound interest holds strongly throughout the “near future,” so that events occurring in the “distant future” are to be shrunken down to equivalent present values by at least the base starting discount factor

$$e^{-\bar{r}T}, \quad (24)$$

which could be quite small if T is large. Second, we can also say that for any two events occurring in the “far-distant future,” their relative discounting should occur at a certainty-equivalent interest rate that declines continuously over time from \bar{r} to $R^* = \min\{r_j\}$, and therefore might be very much lower than \bar{r} .

As a practical matter, whether or not this much lower rate applies to the distant part of any particular project depends on the time horizon of the project relative to

⁶ Although the above proof that $R(t)$ declines monotonically was given here for a discrete probability distribution where $r_j(t) = r_j$ (a constant), the proposition itself is far more general and the proof can be extended to essentially any martingale process where $E[r(t + \tau) | r(t)] = r(t)$.

T. Thus, the paper is suggesting at least the possibility that it may be essential to incorporate declining discount rates into any benefit-cost methodology for evaluating long-term environmental projects.

What is this paper implying about the optimal form of a very long term project under uncertainty—such as, for example, ameliorating the impact of global warming? Other things being equal, the basic proposition should make itself felt by biasing the choice of policy instruments and levels of imposed stringency as if toward what is optimal for the low-interest-rate scenario, because that scenario will weigh more heavily in the expected difference between present-discounted benefits and costs.⁷

The theorem of this paper is phrased in terms of a situation where limiting interest rates exist in different scenarios. But there will also be a modified version to describe situations where low interest rates can persist for a very long time, but not forever. The relevant approximation theorem here will say, roughly speaking, that the results of this paper hold for a mean-reverting process in the limit as the coefficient of reversion goes toward zero. In the world described by such an approximation theorem, the results of this paper will hold for the “far-distant” future centuries of a mean-reverting process with an extremely high degree of persistence, even while they fail to hold for the “far-far-distant” future millennia.

5. CONCLUSION

Uncertainty about future discount rates provides a strong generic rationale for using certainty-equivalent social discount rates that decline over time, from around today’s best average estimate, presumably based on observable market values, down to the smallest imaginable rates for the far distant future.

There are two broad policy implications. First, the paper is suggesting that the decline in certainty-equivalent social discount rates might be a sufficiently significant phenomenon to warrant, at the very minimum, checking out this possibility for any cost-benefit analyses of long-term environmental projects, like mitigating the effects of global warming.

Second, for such long-term environmental evaluations the optimal choice of policy instruments and levels of imposed stringency may well be skewed toward what would be optimal for a low-interest-rate situation because, other things being equal, that situation will carry relatively more weight in determining the expected difference between present discounted benefits and costs.

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⁷ This seems like a roughly accurate description of some of the main findings in Pizer [12].

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