

# Investment Policy for Time-Inconsistent Discounters

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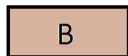
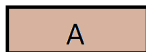
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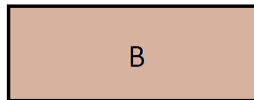
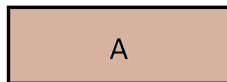
# Like cake?

Which cake do you prefer?

Situation 1



Situation 2



# Public investments/policies with long-term consequences

- Education, child-care
- Research, academic research ("grunnforskning")
- Infrastructure (roads, bridges, tunnels...)
- Extraction of exhaustible/renewable resources
- Conservation, preservation of nature/species
  
- Pollution, emissions, recycling-programs
- **How should we evaluate such projects?**

# The standard approach

- Rae, Jevons, Senior, Bohm-Bawerk: Multiple conflicting psychological factors
- Ramsey (1928):

$$\max \sum_{t=0}^{\infty} D(t) u(c_t).$$

- Samuelson (1937):

$$D(t) = \delta^t = \left( \frac{1}{1+r} \right)^t \approx e^{-rt}.$$

- Koopman's (1960) axiomatic foundation: period-independence, impatience, stationarity, continuity, sensitivity...
- The discount-rate for consumption becomes:

$$\rho \approx \delta + \omega \dot{c}$$

$$\approx 1 + 2 \cdot 4 = 7\%, \text{ where}$$

$$\omega \equiv - \frac{u''(c_0)}{u'(c_0)} c_0.$$

# The standard approach: Norway

- **NOU 1997, 27:109-110:** "En slik konklusjon vil tilsynelatende føre til at få miljøprosjekter eller andre prosjekter med svært langsiktige virkninger blir gjennomført. Det er imidlertid flere forhold som tilsier at [dette] neppe vil gi et slikt resultat"
  - "Dersom senere generasjoner blir rikere enn oss, vil den relative prisen på miljøgoder tendere til å gå opp. Dette følger av at miljøgoder i de fleste tilfeller trolig har høy inntektselastisitet, samtidig som tilbudet vanskelig kan økes. I tillegg vil økt knapphet på miljøgoder føre til at kalkulasjonsprisene går opp"
- Mathematically, given  $u(c_t, e_t)$ , the discount-rate for the e-good is:

$$\begin{aligned}\rho &\approx \delta + \omega_e \dot{e} + \omega_{e,c} \dot{c} < \delta \approx 1\% \text{ if} \\ \dot{e} &< 0 \text{ and} \\ \omega_{e,c} &\equiv -\frac{u_{ec}(c_0, e_0)}{u_e(c_0, e_0)} c_0 < 0.\end{aligned}$$

- "En slik økning i relativ pris bør håndteres gjennom kalkulasjonsprisene..."

# The standard approach: Multiple weaknesses

- Samuelson careful: not suggesting realism or normative
- Koopman's axioms are *very strong*
  - 1 Period-independence: In reality, rising paths often preferred
  - 2 We are required to be impatient
  - 3 We are required to have stationary preferences

# Impatience: *Normative* justifications?

- Standard utilitarianism requires  $\delta = 0$
- Ramsey (1928): "how much of its income should a nation save? ...it is assumed that we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from the weakness of the imagination"
- "a philosophical tradition, stretching from Ramsey 28 to Parfit 84, that has warned us against discounting future utilities without providing serious arguments" (Dasgupta, Barrett, and Maler, 1996).
- "Serious arguments"?
  - 1 Utilitarianism insensitive to distribution?
  - 2 Existence of solutions ("an unfair and absurd test" Phelps 1989)
  - 3 Risk of dying (Blackorby and Donaldson 1977)
- No convincing *normative* argument for  $\delta > 0$

# Impatience: *Empirical* foundation

- A *benevolent planner* would perhaps prefer  $\delta = 0$ 
  - But she has no power in the real world :(
- Today's individuals *do* discount
  - Politicians are individuals - they *do* discount
  - Politicians are accountable/elected by individuals: they *should* and *must* discount
- As an economist: I must help them out!



## Stationarity: Lack of *empirical* foundation

- But with this motivation, we need to understand "time preferences"
- Stationarity is technically convenient, but lacks both theoretical and empirical foundations
- **Intuitively:** The difference between  $t$  and  $t + 1$  vanishes as  $t$  grows (Strotz 1955). Human senses compares relative differences (this is not "irrational")
- **Empirically:** Individuals are not consistent over time: Eisenhauer and Ventura (2006), Angeletos et al, O'Donoghue and Rabin (1999): hyperbolic individuals will show exactly the low IRA participation we observe
- **Experimentally:** Viscusi and Huber (2006), Kirby and Marakovic (1995), Benhabib, Bisin and Schollter (2010), Ainslie (1992), Kirby and Herrnstein (1995), Thaler
- **Neuroscience:** McClure et al (2009), McClure et al (2004)
- **Governments:** In practice, policymakers have begun applying lower discount rates to long-term, intergenerational projects (Bazerlon and Smetters 1999)

# Stationarity: Lack of *theoretical* foundation

- Thoughtful parents, Arrow (1973), Dasgupta (1974), Barro and Becker (1989):

$$w_t = u(c_t) + \delta w_{t+1}$$

- Recursively,

$$w_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_{\tau})$$

- But suppose:

$$w_t = u(c_t) + \delta_1 w_{t+1} + \delta_2 w_{t+2} + \dots$$

- Then, the effective discount rate decreases in  $t$  (Saez-Marti and Weibull, 2005 - and Harstad 1999)

# Actual time preferences: What are they?

- A variety of estimates, but exponential discounting rejected.

$$w_t = \sum_{\tau=t}^{\infty} (\prod_{s=t}^{\tau} \delta_{s-t}) u_{\tau}(c_{\tau})$$

- Hyperbolic discounting:

$$D(t) = \frac{1}{1 + kt}, k > 0.$$

- Phelps and Pollak (1968) argued for "imperfect altruism", where  $\beta < 1$ :

$$w_t = u(c_t) + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u(c_{\tau}).$$

- Laibson (1994 - 2030) etc adopts this function also to within-lifetime choices. Many names:  $(\beta, \delta)$ -discounting, quasi-geometric discounting, quasi-exponential discounting, quasi-hyperbolic discounting, hyperbolic discounting

# Consequences for project evaluation

- Discount benefits/costs at  $t$  at a lower rate if  $t$  is large
- With hyperbolic discounting, implement project if:

$$\begin{aligned}\Delta u_0 + \frac{1}{1+kt} \Delta u_t &> 0 \Rightarrow \\ \frac{1}{1+\bar{r}} &= \sqrt[t]{\frac{1}{1+kt}} \\ &\rightarrow 1 \text{ as } t \rightarrow \infty.\end{aligned}$$

- With  $(\beta, \delta)$ -discounting, implement project if:

$$\begin{aligned}\Delta u_0 + \beta \delta^t \Delta u_t &> 0 \Rightarrow \\ \frac{1}{1+\bar{r}} &= \delta \sqrt[t]{\beta} \text{ or} \\ &\rightarrow \delta \text{ as } t \rightarrow \infty.\end{aligned}$$

# Consequences for stationarity

- Costly today? Better to postpone until tomorrow!
  - People save too little (subsidize saving: Krusell et al 2010)
  - Retire too early (Diamond and Koszegi 2003)
  - Reason for obesity (Scharff 2009)
  - Smoke too much (tax tobacco more: Gruber and Koszegi 2001, 2004)
  - Governments delay projects (moon-landing), etc
- Great plans for the future postponed

# Consequences for commitment

- With time inconsistency, pre-commitment is beneficial:
  - Saving plans (costly to exit). Thaler and Benartzi's (2004) "Save More Tomorrow" program (SMarT)
  - Annual gym-membership (DellaVigna and Malmendier 2006) or appointments with friends
  - Establish rules: No deficit (handlingsregelen)
  - Investments in technologies...

# A simple model with technology

- Suppose  $w_0 = \sum_{\tau=0}^{\infty} (\prod_{s=0}^{\tau} \delta_s) u_{\tau}(c_{\tau})$ ,  $\delta_1 < \delta_2 < \dots$
- In period 1, the government pollutes  $e$ , causing harm  $-e$  in period 2:

$$u_e(e, a) = \delta_1$$

- In period 0, we would have preferred a smaller  $e$ , given by:

$$u_e(e, a) = \delta_2 > \delta_1$$

- Invest in technology  $a$  to influence future decision?
- Answer: If  $u_{ea} = 0$ , invest until:

$$c'(a) = \delta_1 u_a(e, a).$$

- More generally: Invest more in substitutes, less in complements:

$$\begin{aligned} c'(a) &= \delta_1 \left[ u_a + \frac{u_{ea}}{u_{ee}} (\delta_2 - \delta_1) \right] \\ &= u_a(e, a) \left[ \delta_1 + \frac{u_{ea} u_e}{u_a u_{ee}} (\delta_2 - \delta_1) \right] \end{aligned}$$

# Adjust the discount factor/rate?

- Discount factor:

$$c'(a) = \delta_a u_a \text{ where}$$
$$\delta_a = \delta_1 \left[ 1 + \frac{u_{ea} u_e}{u_a u_{ee}} \left( \frac{\delta_2}{\delta_1} - 1 \right) \right]$$

- With quasi-hyperbolic discounting:

$$\delta_a = \beta \delta \left[ 1 + \frac{u_{ea} u_e}{u_a u_{ee}} \left( \frac{1}{\beta} - 1 \right) \right].$$

- With perfect substitutes,  $u = u(g + R)$ :

$$\delta_a = \delta > \beta \delta = \delta_1.$$

- With perfect complements (Cobb-Douglas):

$$\delta_a = \delta (2\beta - 1) < \beta \delta = \delta_1 < \delta = \delta_2.$$



# Subsidize/tax private investments?

- With subsidy  $s$  (or tax, if  $s < 0$ ) private invest until:

$$c'(a) = \delta_1 (1 + s) u_a,$$

- The optimal  $s$  is:

$$s_a = \frac{u_{ea}u_e}{u_a u_{ee}} \left( \frac{\delta_2}{\delta_1} - 1 \right).$$

- With quasi-hyperbolic discounting:

$$s_a = \frac{u_{ea}u_e}{u_a u_{ee}} \left( \frac{1}{\beta} - 1 \right).$$

- For perfect substitutes,  $u = u(e + a)$ :

$$s_a = \frac{1}{\beta} - 1 > 0.$$

- For perfect complements (Cobb-Douglas):

$$s_a = 1 - \frac{1}{\beta} < 0.$$

# Conclusions

- 1 Intuitively, empirically, and experimentally: Discount rates decline with the time horizon
- 2 This should be reflected in public CBA
- 3 Long-term benefits/costs should be discounted at a lower rate
- 4 Time inconsistency is a consequence and concern; not a critique
- 5 Pre-commitments are beneficial
- 6 Many public projects permit such commitments
  - 1 Investments in substitutes to future bads, or complements to future goods, should be subsidized or discounted at lower rates
  - 2 Investments in substitutes to future goods, or complements to future bads, should be taxed or discounted at higher rates